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TABLES OF THE VELOCITY OF SOUND
IN PURE WATER AND SEA WATER FOR
USE IN. ECHO-SOUNDING AND SOUND-
RANGING.

Hydrographic Department, Admiralty.

H.D. 282.

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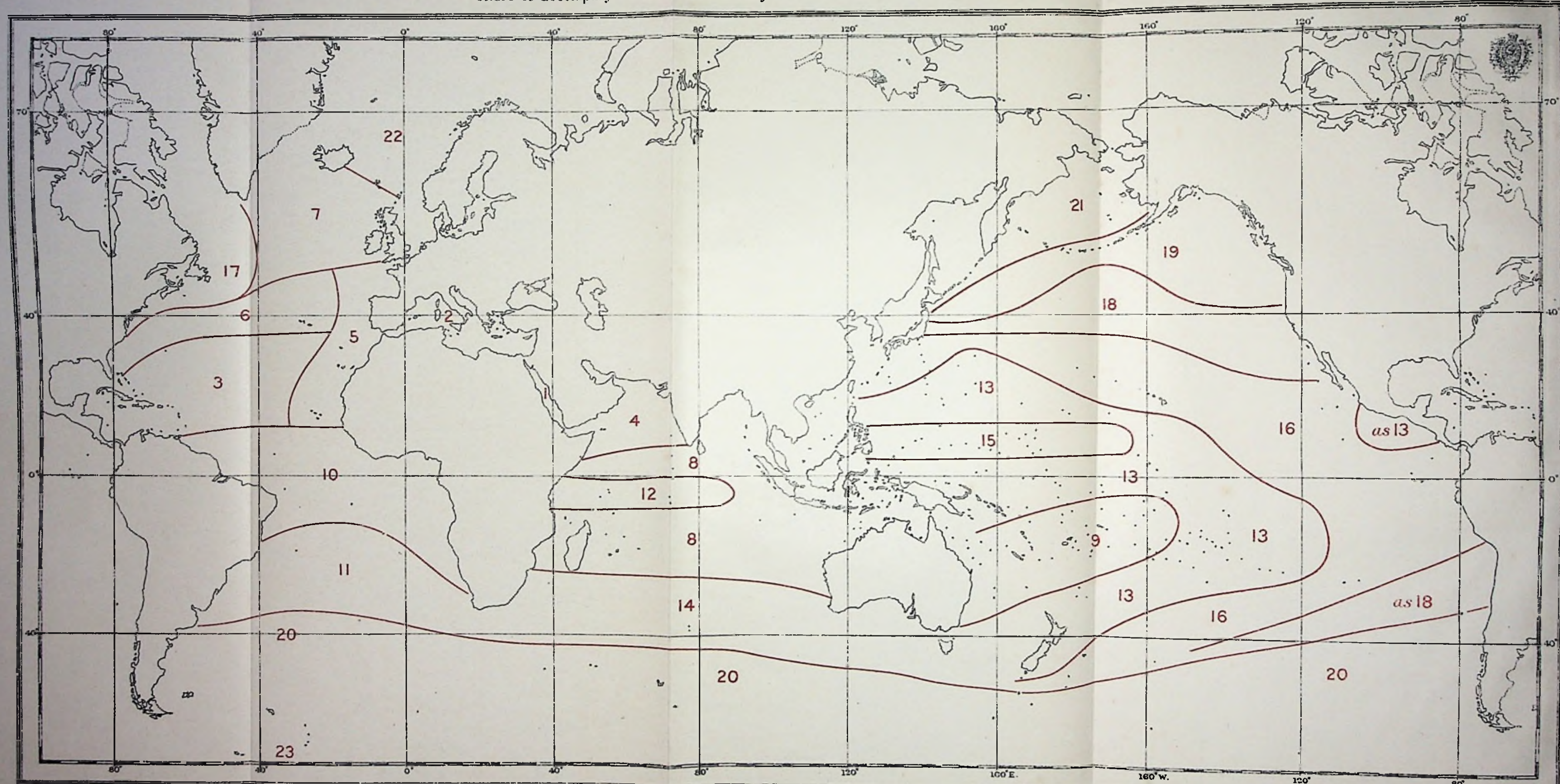
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ECHO-SOUNDING REGIONS

Chart to accompany "Tables for the Velocity of Sound in Pure Water and Sea Water 1926."



Prepared by the Hydrographic Dept of the Admiralty, 28th Oct. 1926, under the Superintendence of Captain H.P. Douglas, C.M.G., R.N., Hydrographer.
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PREFACE.

The application of submarine sound signals to the measurement of depths and ranges has made such rapid progress during the last ten years that an accurate knowledge of the velocity of sound in water has become imperative.

The velocity can either be determined directly or it can be calculated on theoretical grounds from certain properties of water which have been determined in the laboratory.

The direct measurement is a difficult and costly operation. It necessitates the fixing of the position of a number of submarine hydrophones at intervals of some miles and at such a depth and at such a distance from the shore that disturbance by echoes is not to be feared, in a locality where the temperature and salinity are as nearly as may be uniform everywhere and where there is a period of almost complete slack water at every tide. The hydrophones must be connected by means of insulated cables with recording instruments capable of measuring short intervals of time with an accuracy of the order of one-thousandth of a second. Given such an equipment and suitable weather it is possible to make satisfactory measurements of the velocity under the conditions of temperature and salinity obtaining at the position in question. To cover all the conditions likely to be encountered in the sea would require at least ten such stations and one at least of them would have to be situated within the arctic circle.

The determination of the velocity by means of comparison of echo-soundings with those obtained by means of the Lucas and other similar machines is out of the question on account of the uncertainty which the drift of the ship introduces into all wire soundings.

The sounding tables and tables of the velocity of sound contained in this book are founded on the indirect method which is fully explained in Part III.

The calculations have been made and the tables prepared by Mr. D. J. Matthews in co-operation with the Hydrographer of the Navy and the Director of Scientific Research (Admiralty), and are now published with the approval of the Board of Admiralty.

H. P. DOUGLAS,
Hydrographer of the Navy.

INTRODUCTION.

The velocity of sound in sea water depends upon the temperature, salinity and pressure, and before a sounding or a range measured by acoustic methods can be calculated these quantities must be known at a sufficient number of points. The calculation is always a somewhat lengthy operation, and in many cases the temperature and salinity will not be known with sufficient accuracy at the time. Twenty-three average sounding tables have therefore been prepared which may be used for preliminary work without very serious error; they are given in Part I.

Tables of the velocity of sound in sea water under various conditions will be found in Part II, together with instructions for their use in the preparation of accurate sounding tables and the calculation of ranges.

The method used in the preparation of these tables is given in Part III.

TABLE 1—continued.

Depth.		9.			10.			11.			12.		
		Fiji Is. REGION.			EQUATORIAL ATLANTIC OCEAN.			CENTRAL SOUTH ATLANTIC OCEAN.			INDIAN OCEAN COUNTER CURRENT.		
		10° S. lat., 180° E. long.			5° S. lat., 32° W. long.			20° S. lat., 20° E. long.			5° S. lat., 51° E. long.		
Fms.	Ms.	t° C.	Sounding Velocity per sec.		t° C.	Sounding Velocity per sec.		t° C.	Sounding Velocity per sec.		t° C.	Sounding Velocity per sec.	
			Fms.	Ms.		Fms.	Ms.		Fms.	Ms.		Fms.	Ms.
0	0	27.5	—	—	25.8	—	—	24.5	—	—	28.1	—	—
55	100	27.2	—	—	25.1	—	—	21.3	—	—	20.0	—	—
109	200	22.5	840	1536	11.9	834	1525	16.0	832	1522	14.4	832	1522
219	400	11.6	834	1525	8.5	825	1509	10.7	827	1513	10.3	826	1511
328	600	7.8	828	1515	6.1	821	1501	6.4	823	1505	8.3	823	1505
437	800	5.8	825	1508	4.4	818	1496	4.3	820	1500	7.0	821	1501
547	1000	4.5	822	1503	3.8	816	1492	3.8	818	1495	6.1	820	1499
656	1200	3.9	820	1500	—	815	1491	—	816	1493	5.3	819	1498
766	1400	3.4	819	1498	—	815	1490	—	816	1492	4.5	819	1497
875	1600	3.0	818	1496	—	815	1490	—	815	1491	4.0	818	1496
984	1800	2.8	818	1496	—	815	1490	—	815	1491	3.4	818	1496
1094	2000	2.6	818	1495	3.3	815	1491	3.0	815	1491	2.6	818	1495
1203	2200	—	818	1495	—	815	1491	—	816	1492	—	818	1496
1312	2400	—	818	1495	—	816	1492	—	816	1492	—	818	1496
1422	2600	—	818	1496	—	816	1493	—	816	1493	—	818	1496
1531	2800	—	819	1497	—	817	1494	—	817	1494	—	818	1496
1641	3000	2.2	819	1497	2.3	818	1495	2.6	818	1495	1.9	819	1497
1750	3200	—	819	1498	—	818	1496	—	818	1496	—	819	1498
1859	3400	—	819	1499	—	819	1497	—	819	1497	—	820	1499
1969	3600	—	820	1500	—	819	1498	—	819	1498	—	820	1500
2078	3800	—	—	—	—	820	1499	—	820	1500	—	821	1501
2187	4000	—	—	—	1.3	820	1500	1.6	821	1501	1.6	821	1502
2297	4200	—	—	—	—	821	1501	—	821	1502	—	822	1503
2406	4400	—	—	—	—	821	1502	—	822	1503	—	822	1503
2515	4600	—	—	—	0.7	822	1504	—	822	1504	—	822	1504
2625	4800	—	—	—	—	—	—	—	823	1505	—	823	1505
2734	5000	—	—	—	—	—	—	1.2	824	1507	1.2	824	1506

TABLE 1—continued.

Depth.		21. BERING SEA. 55° N. lat., 170° W. long.			22. NORWEGIAN SEA. 69° N. lat., 5° E. long.			23. ANTARCTIC OCEAN. 54° S. lat., 31° W. long.		
		t° C.	Sounding Velocity per sec.		t° C.	Sounding Velocity per sec.		t° C.	Sounding Velocity per sec.	
			Fms.	Ms.		Fms.	Ms.		Fms.	Ms.
0	0	3.5	—	—	9.7	—	—	-1.1	—	—
55	100	3.3	—	—	5.2	—	—	-1.2	—	—
109	200	2.8	799	1461	4.3	806	1473	0.4	789	1443
219	400	3.5	800	1463	3.6	804	1471	1.1	792	1448
328	600	3.4	801	1465	3.1	804	1470	1.2	793	1451
437	800	3.3	802	1467	1.6	804	1470	0.9	795	1454
547	1000	3.0	803	1469	0.1	802	1467	0.7	796	1456
656	1200	2.8	804	1470	—	802	1467	—	797	1458
766	1400	2.5	805	1472	—	802	1467	—	798	1460
875	1600	2.3	806	1473	—	802	1467	0.4	800	1463
984	1800	2.2	807	1475	—	803	1468	—	801	1465
1094	2000	2.1	807	1476	-0.9	803	1469	0.2	802	1466
1203	2200	2.0	808	1478	—	804	1470	—	803	1468
1312	2400	2.0	809	1479	—	804	1471	—	803	1469
1422	2600	1.9	810	1481	—	805	1472	—	804	1471
1531	2800	1.9	810	1482	—	806	1474	—	805	1472
1641	3000	1.8	812	1484	-1.0	807	1476	0.1	806	1474
1750	3200	—	—	—	—	—	—	—	807	1476
1859	3400	—	—	—	—	—	—	—	808	1477
1969	3600	—	—	—	—	—	—	—	809	1479
2078	3800	—	—	—	—	—	—	—	809	1480
2187	4000	—	—	—	—	—	—	-0.1	810	1482
2297	4200	—	—	—	—	—	—	—	811	1484
2406	4400	—	—	—	—	—	—	—	812	1485
2515	4600	—	—	—	—	—	—	—	813	1487
2625	4800	—	—	—	—	—	—	—	814	1489
2734	5000	—	—	—	—	—	—	—	815	1490
2844	5200	—	—	—	—	—	—	—	816	1492
2953	5400	—	—	—	—	—	—	—	817	1494
3062	5600	—	—	—	—	—	—	-0.4	818	1496

PART II.

THE VELOCITY OF SOUND IN SEA WATER AND THE
CALCULATION OF SOUNDINGS.

The velocity of sound in sea-water depends upon the temperature, salinity and pressure. In the following tables—

Table 2 gives the velocity in metres per second under atmospheric pressure in an average sea-water of salinity 34.85 parts per thousand by weight. This salinity corresponds to a density of 1.02800 at $0^{\circ}/4^{\circ}$ C., that is, at 0° C. compared with pure water at 4° C., its temperature of maximum density. In oceanographical work it is usual to use for this the abbreviation $\sigma_0 = 28.00$; in the same way $\sigma_0 = 5.00$ means that the density at $0^{\circ}/4^{\circ}$ C. is 1.00500.

Table 3 gives the corrections to be applied to the velocities in Table 2 on account of salinity or density at $0^{\circ}/4^{\circ}$ C. for salinities more than 30 per thousand. The sign of the correction should be noted. The velocities for lower salinities which are seldom required are given in Table 6 and can be corrected by interpolation.

Table 4 gives the corrections for depth, which are really corrections on account of pressure calculated on the assumptions that the latitude is 45° and that the water is everywhere at 0° C. and has a salinity of 34.85 per thousand. The corresponding pressure in decibars is also given so that the velocity under any pressure may be taken out by interpolation if desired. It should be noted that atmospheric pressure is considered to be zero throughout.

Table 5 gives the corrections on account of the change of the force of gravity with latitude and is to be used when Table 4 is entered with depth but not if it is entered with pressure.

The assumptions made in calculating Table 4 are not quite correct, and in particular the temperature in a column of water in the seas is usually higher than 0° C., and consequently the depth at which a given pressure occurs in 45° lat. is generally greater than that shown. The resulting error in velocity is, however, so small that it has not been considered necessary to add another table of corrections.

In the same way no account has been taken of the fact that the pressure correction is not the same for all salinities, since salinities differing widely from 34.85 per thousand do not occur at great depths.

The use of the tables is explained in the following examples :—

Example 1.—Required the velocity at the surface in water of 17.5° C. and 34.85 salinity. Table 2 gives 1511.3 m./sec.

Example 2.—Required the velocity at the surface in water at 10.15° C. and 33.9 salinity—

By Table 2	-	-	-	-	1487.0 m/sec.
"	3	-	-	-	- 1.0 "
					<hr/>
					1486.0 "

Example 3.—Required the velocity at 12.5° C. in water of $\sigma_0 = 30.6$ at a depth of 3700 m., in latitude 35° —

By Table 2	-	-	-	-	1495.4 m/sec.
"	3	-	-	-	3.8 "
"	4	-	-	-	65.9 "
"	5	-	-	-	- 0.1 "
					<hr/>
					1565.0 "

Example 4.—Required the velocity at 2° C. in water of 34.5 salinity under a pressure of 3700 decibars—

By Table 2	-	-	-	-	1454.2 m/sec.
"	3	-	-	-	- 0.3 "
"	4	-	-	-	65.8 "
					<hr/>
					1519.7 "

THE METHOD OF CALCULATING A SOUNDING.

The examples given above explain the method of calculating the velocity under any definite conditions, that is, the horizontal velocity over a short distance in which temperature, salinity and pressure can be regarded as unchanging. In an echo sounding the sound wave passes vertically downwards under continually increasing pressure and very often in water of decreasing temperature and salinity, and it is obvious that the velocity will change from depth to depth according to a complicated law which can be expressed in a formula susceptible to mathematical treatment only in exceptional cases. It is therefore necessary to calculate the mean horizontal velocity in each water layer between the surface and the required depth. The mean of these means is the mean vertical velocity over the distance in question, the "sounding velocity." There is, however, more than one way of calculating a mean. A little reflection will show that the correct procedure would be to calculate the time required for the sound to pass through each of a sufficiently large number of equal layers, to add these, and to divide by the total distance; the result would be the harmonic mean, the true sounding velocity. The calculation is very laborious and tests have shown that the arithmetical mean does not generally differ from the harmonic mean by more than 0.2 m./sec.; the arithmetical mean has been used throughout in this work.

Two methods of calculating a sounding table from known temperatures and salinities are shown in Table 7, which is

founded on observations made by the German Antarctic Expedition in the *Deutschland* in 1911. The depth is given in column 1, the temperature and salinity in columns 2 and 3, the larger number of these below 1,000 m. having been interpolated. In column 4 is the horizontal velocity calculated by the methods already explained in detail, in column 5 the mean horizontal velocity in each layer 200 m. thick, and in column 6 the mean of these last from the surface down to each 200 m. level; for instance, against 600 m. is 1,517.9 m./sec., which is the mean of the first three velocities in column 5 and is the correct velocity to use if the depth is about 600 m.

In nearly every case the arithmetical work may be much shortened by using the procedure explained below. In the present example the mean temperature and salinity has been calculated in each layer of 200 m. thickness down to a depth of 2,000 m. and below that in each layer 1,000 m. thick as shown in columns 7 and 8. The mean corresponding velocity is then calculated for each layer as shown in column 9, and from these last the sounding velocities down to each 200 m. level in the upper 2,000 m. and to each 1,000 m. level at greater depths. They are shown unbracketed in column 10. The bracketed velocities at 2,200 m., 2,400 m. and so on have been interpolated graphically; linear interpolation often gives rise to serious errors. It is a common rule in oceanography to observe temperature and salinity at frequent intervals down to a depth of 1,000 m. and then at 1,500 m., 2,000 m., 3,000 m. and so on; in such a case the velocity should be calculated for the two 500 m. layers. Care should be taken to use the correction for depth for the middle of each layer.

The velocities in columns 6 and 10 agree closely and the second method is recommended for general use.

In the upper layers the rapid variation of the temperature makes the sounding velocity uncertain, but the resulting error in the sounding is usually inconsiderable. If, for instance, the depth is 400 m. an error of 5 m./sec. in the velocity gives rise to an error of about one and one-third metre in the sounding.

CALCULATIONS FOR SOUND RANGING.

The calculation of a range determined by acoustic methods is comparatively simple. It is only necessary to take out the velocity corresponding to the average temperature and salinity and correct it for the depth at which it is believed the sound travels. The chief cause of error will be the use of incorrect temperatures and salinities.

TABLE 2.

Velocity of Sound (in metres per second) in Seawater.

$$\sigma_0 = 28.00. \quad S = 34.85\%_{00}.$$

at the Surface under Atmospheric Pressure.

t °C.	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9
—2°	1435.9	—	—	—	—	—	—	—	—	—
—1°	1440.6	1440.1	1439.7	1439.2	1438.7	1438.3	1437.8	1437.3	1436.9	1436.4
—0°	1445.2	1444.8	1444.3	1443.8	1443.4	1442.9	1442.5	1442.0	1441.5	1441.1
0°	1445.2	1445.7	1446.1	1446.6	1447.1	1447.5	1448.0	1448.4	1448.9	1449.3
1°	1449.8	1450.2	1450.7	1451.1	1451.6	1452.0	1452.5	1452.9	1453.3	1453.8
2°	1454.2	1454.7	1455.1	1455.5	1456.0	1456.4	1456.9	1457.3	1457.7	1458.2
3°	1458.6	1459.0	1459.4	1459.9	1460.3	1460.7	1461.2	1461.6	1462.0	1462.4
4°	1462.9	1463.3	1463.7	1464.1	1464.5	1465.0	1465.4	1465.8	1466.2	1466.6
5°	1467.0	1467.4	1467.9	1468.3	1468.7	1469.1	1469.5	1469.9	1470.3	1470.7
6°	1471.1	1471.5	1471.9	1472.3	1472.7	1473.1	1473.5	1473.9	1474.3	1474.7
7°	1475.1	1475.5	1475.9	1476.3	1476.7	1477.1	1477.4	1477.8	1478.2	1478.6
8°	1479.0	1479.4	1479.8	1480.1	1480.5	1480.9	1481.3	1481.7	1482.0	1482.4
9°	1482.8	1483.2	1483.5	1483.9	1484.3	1484.6	1485.0	1485.4	1485.8	1486.1
10°	1486.5	1486.8	1487.2	1487.6	1487.9	1488.3	1488.7	1489.0	1489.4	1489.8
11°	1490.1	1490.5	1490.8	1491.2	1491.5	1491.9	1492.2	1492.6	1492.9	1493.3
12°	1493.6	1494.0	1494.3	1494.7	1495.0	1495.4	1495.7	1496.0	1496.4	1496.7
13°	1497.1	1497.4	1497.7	1498.1	1498.4	1498.7	1499.1	1499.4	1499.7	1500.1
14°	1500.4	1500.7	1501.0	1501.4	1501.7	1502.0	1502.3	1502.7	1503.0	1503.3

TABLE 2—continued.

t °C.	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9
15°	1503.6	1503.9	1504.3	1504.6	1504.9	1505.2	1505.5	1505.8	1506.1	1506.5
16°	1506.8	1507.1	1507.4	1507.7	1508.0	1508.3	1508.6	1508.9	1509.2	1509.5
17°	1509.8	1510.1	1510.4	1510.7	1511.0	1511.3	1511.6	1511.9	1512.2	1512.5
18°	1512.8	1513.1	1513.4	1513.7	1514.0	1514.3	1514.5	1514.8	1515.1	1515.4
19°	1515.7	1516.0	1516.3	1516.5	1516.8	1517.1	1517.4	1517.7	1517.9	1518.2
20°	1518.5	1518.8	1519.1	1519.3	1519.6	1519.9	1520.1	1520.4	1520.7	1521.0
21°	1521.2	1521.5	1521.8	1522.0	1522.3	1522.6	1522.8	1523.1	1523.4	1523.6
22°	1523.9	1524.2	1524.4	1524.7	1524.9	1525.2	1525.5	1525.7	1526.0	1526.2
23°	1526.5	1526.7	1527.0	1527.3	1527.5	1527.8	1528.0	1528.3	1528.5	1528.8
24°	1529.0	1529.3	1529.5	1529.8	1530.0	1530.3	1530.5	1530.7	1531.0	1531.2
25°	1531.5	1531.7	1532.0	1532.2	1532.4	1532.7	1532.9	1533.2	1533.4	1533.6
26°	1533.9	1534.1	1534.3	1534.6	1534.8	1535.1	1535.3	1535.5	1535.8	1536.0
27°	1536.2	1536.5	1536.7	1536.9	1537.1	1537.4	1537.6	1537.8	1538.1	1538.3
28°	1538.5	1538.7	1539.0	1539.2	1539.4	1539.6	1539.9	1540.1	1540.3	1540.5
29°	1540.8	1541.0	1541.2	1541.4	1541.6	1541.8	1542.1	1542.3	1542.5	1542.7
30°	1542.9	—	—	—	—	—	—	—	—	—

TABLE 3.

Corrections to Table 2 on account of Salinity or Density 0°/4° C.

Subtract Correction.		Add Correction.		° C.						
Salinity.	σ_0	Salinity.	σ_0	0°	5°	10°	15°	20°	25°	30°
34.7	27.89	34.90	28.05	0	0	0	0	0	0	0
.6	.80	35.0	.13	0.1	0.1	0.1	0.1	0.1	0.1	0.1
.5	.72	.1	.21	0.3	0.3	0.3	0.2	0.2	0.2	0.2
.4	.64	.2	.29	0.4	0.4	0.4	0.4	0.4	0.3	0.3
.3	.56	.3	.37	0.5	0.5	0.5	0.5	0.5	0.4	0.4
.2	.48	.4	.45	0.7	0.7	0.6	0.6	0.6	0.5	0.5
.1	.40	.5	.53	0.9	0.9	0.8	0.7	0.7	0.7	0.7
.0	.32	.6	.61	1.1	1.1	0.9	0.8	0.8	0.8	0.8
33.9	.24	.7	.69	1.2	1.2	1.0	0.9	0.9	0.9	0.9
.8	.16	.8	.77	1.3	1.2	1.1	1.1	1.1	1.0	1.0
.7	.08	.9	.85	1.4	1.3	1.3	1.2	1.2	1.1	1.1
.6	.00	36.0	.94	1.5	1.5	1.4	1.3	1.3	1.2	1.2
.5	26.92	.1	29.01	1.6	1.6	1.5	1.4	1.4	1.3	1.3
.4	.84	.2	.10	1.7	1.7	1.6	1.5	1.5	1.4	1.4
.3	.75	.3	.18	1.9	1.9	1.8	1.6	1.6	1.5	1.5
.2	.67	.4	.26	2.0	2.0	1.9	1.7	1.7	1.6	1.6
.1	.59	.5	.34	2.1	2.1	2.0	1.9	1.8	1.7	1.7
.0	.51	.6	.42	2.3	2.2	2.1	2.0	1.9	1.8	1.8
32.9	.43	.7	.50	2.4	2.4	2.3	2.1	2.1	2.0	1.9
.8	.35	.8	.58	2.5	2.5	2.4	2.2	2.2	2.1	2.0
.7	.27	.9	.66	2.7	2.6	2.5	2.3	2.3	2.2	2.1
.6	.20	37.0	.74	2.8	2.7	2.6	2.4	2.4	2.3	2.2
.5	.11	.1	.82	2.9	2.8	2.8	2.6	2.5	2.4	2.3
.4	.03	.2	.90	3.1	3.0	2.9	2.7	2.6	2.5	2.4
.3	25.95	.3	.98	3.2	3.1	3.0	2.8	2.7	2.6	2.5
.2	.87	.4	30.06	3.3	3.2	3.1	2.9	2.8	2.7	2.6
.1	.79	.5	.14	3.5	3.4	3.3	3.0	2.9	2.8	2.7
.0	.71	.6	.22	3.6	3.5	3.4	3.2	3.1	2.9	2.8
31.9	.63	.7	.30	3.7	3.6	3.5	3.3	3.2	3.0	2.9
.8	.55	.8	.38	3.9	3.8	3.6	3.4	3.3	3.1	3.0
.7	.45	.9	.46	4.0	3.9	3.7	3.5	3.4	3.3	3.2
.6	.39	38.0	.54	4.1	4.0	3.9	3.6	3.5	3.4	3.3
.5	.31	.1	.63	4.2	4.1	4.0	3.7	3.6	3.5	3.4
.4	.23	.2	.71	4.4	4.3	4.1	3.8	3.7	3.6	3.5
.3	.15	.3	.79	4.5	4.4	4.2	4.0	3.8	3.7	3.6
.2	.07	.4	.87	4.6	4.5	4.4	4.1	4.0	3.8	3.7
.1	24.99	.5	.95	4.7	4.6	4.5	4.3	4.1	3.9	3.8
.0	.91	.6	31.03	4.9	4.8	4.6	4.3	4.2	4.0	3.9
30.9	.82	.7	.11	5.0	4.9	4.7	4.4	4.3	4.1	4.0
.8	.74	.8	.19	5.2	5.1	4.8	4.5	4.4	4.2	4.1
.7	.66	.9	.27	5.3	5.2	5.0	4.7	4.5	4.3	4.2
.6	.58	39.0	.35	5.4	5.3	5.1	4.8	4.6	4.4	4.3
.5	.50	.1	.43	5.6	5.4	5.2	4.9	4.7	4.5	4.4
.4	.42	.2	.51	5.7	5.6	5.3	5.0	4.9	4.7	4.5
.3	.34	.3	.60	5.8	5.7	5.4	5.1	5.0	4.8	4.6
.2	.26	.4	.68	5.9	5.8	5.5	5.2	5.1	4.9	4.7
.1	.18	.5	.76	6.1	5.9	5.7	5.4	5.2	5.0	4.8

TABLE 3—continued.

Subtract Correction.		Add Correction.		° C.						
Salinity.	σ_0	Salinity.	σ_0	0°	5°	10°	15°	20°	25°	30°
30.0	24.10	39.6	31.84	6.2	6.1	5.8	5.5	5.3	5.1	4.9
		.7	.92	6.4	6.2	5.9	5.6	5.4	5.2	5.0
		.8	32.00	6.5	6.3	6.0	5.7	5.5	5.3	5.1
		.9	.08	6.6	6.4	6.1	5.8	5.6	5.4	5.2
		40.0	.17	6.8	6.6	6.2	5.9	5.7	5.5	5.3
		.1	.24	6.9	6.7	6.4	6.0	5.8	5.6	5.4
		.2	.32	7.0	6.8	6.5	6.1	5.9	5.7	5.5
		.3	.40	7.2	7.0	6.6	6.2	6.0	5.8	5.6
		.4	.48	7.3	7.1	6.7	6.4	6.2	6.0	5.7
		.5	.57	7.4	7.2	6.8	6.5	6.3	6.1	5.8
		.6	.65	7.5	7.3	6.9	6.6	6.4	6.2	5.9
		.7	.73	7.7	7.5	7.1	6.7	6.5	6.3	6.0
		.8	.81	7.8	7.6	7.2	6.8	6.6	6.4	6.1
		.9	.89	7.9	7.7	7.3	6.9	6.7	6.5	6.2
		41.0	.97	8.0	7.8	7.4	7.0	6.8	6.6	6.3

TABLE 4.

Corrections to be added to the Velocities in Table 2 on account of Pressure or Depth.

The Depth-corrections are calculated for 45° Lat. and a uniform Temperature of 0° C. and Salinity 34.85 per thousand.

Depth : Metres. } 0	100	200	300	400	500	600	700	800	900	
Pressure : Decibars. } 0	100.8	201.7	302.6	403.6	504.6	605.7	706.8	808.0	909.2	
t °C.										
-2	0	1.8	3.6	5.5	7.3	9.1	10.9	12.7	14.6	16.4
5	0	1.8	3.6	5.5	7.3	9.1	10.9	12.7	14.5	16.4
10	0	1.8	3.6	5.4	7.2	9.0	10.8	12.6	14.5	16.3
15	0	1.8	3.6	5.4	7.2	9.0	10.8	12.6	14.4	16.2
20	0	1.8	3.6	5.4	7.2	9.0	10.8	12.6	14.4	16.2
25	0	1.8	3.6	5.4	7.3	9.1	10.9	12.7	14.5	16.3

Depth : Metres. } 1000	1100	1200	1300	1400	1500	1600	1700	1800	1900	
Pressure : Decibars. } 1010.5	1111.8	1213.2	1314.7	1416.2	1517.7	1619.3	1720.9	1822.5	1924.2	
t °C.										
-2	18.2	20.0	21.8	23.6	25.4	27.2	29.1	30.9	32.7	34.5
5	18.2	20.0	21.8	23.6	25.4	27.2	29.0	30.8	32.6	34.4
10	18.1	19.9	21.7	23.5	25.3	27.1	28.9	30.7	32.5	34.3
15	18.0	19.8	21.6	23.4	25.2	27.0	28.8	30.5	32.3	34.1
20	18.0	19.8	21.7	23.4	25.2	27.0	28.7	30.5	32.3	34.1
25	18.1	19.9	21.7	23.5	25.3	27.1	28.9	30.8	32.6	34.4

TABLE 4—continued.

Depth : Metres. }	2000	2100	2200	2300	2400	2500	2600	2700	2800	2900
Pressure : Decibars. }	2026.0	2127.8	2229.6	2331.5	2433.5	2535.5	2637.5	2739.6	2841.7	2943.9
t °C.										
-2	36.3	38.1	39.9	41.7	43.6	45.4	47.2	49.0	50.8	52.6
-1	36.3	38.1	39.9	41.8	43.6	45.4	47.2	49.0	50.8	52.6
0	36.3	38.1	39.9	41.8	43.6	45.4	47.2	49.0	50.8	52.6
1	36.3	38.1	39.9	41.7	43.6	45.4	47.2	49.0	50.8	52.6
2	36.3	38.1	39.9	41.7	43.5	45.4	47.2	49.0	50.8	52.6
3	36.3	38.1	39.9	41.7	43.5	45.3	47.1	48.9	50.7	52.5
4	36.3	38.1	39.9	41.7	43.5	45.3	47.1	48.9	50.7	52.5
5	36.2	38.1	39.9	41.7	43.5	45.3	47.0	48.8	50.6	52.4
10	36.1	37.8	39.6	41.4	43.2	44.9	46.7	48.5	50.3	52.0
15	35.9	37.7	39.4	41.3	43.0	44.8	46.6	48.4	50.1	51.8
20	35.9	37.6	39.4	41.2	42.9	44.7	46.5	48.2	50.0	51.8
25	36.2	37.7	39.5	41.2	43.0	44.8	46.6	48.3	50.1	51.8

Depth : Metres. }	3000	3100	3200	3300	3400	3500	3600	3700	3800	3900
Pressure : Decibars. }	3046.1	3148.4	3250.7	3353.1	3455.5	3558.0	3660.5	3763.0	3865.6	3968.3
t °C.										
-2	54.4	56.2	58.0	59.8	61.6	63.4	65.2	67.0	68.8	70.6
-1	54.4	56.2	58.0	59.8	61.6	63.4	65.2	67.0	68.8	70.6
0	54.4	56.2	58.0	59.8	61.6	63.4	65.2	67.0	68.8	70.6
1	54.4	56.2	58.0	59.8	61.6	63.4	65.2	66.9	68.7	70.5
2	54.4	56.2	57.9	59.7	61.5	63.3	65.1	66.9	68.7	70.5
3	54.3	56.1	57.9	59.7	61.5	63.3	65.0	66.8	68.6	70.4
4	54.3	56.1	57.8	59.6	61.4	63.2	65.0	66.8	68.5	70.3
5	54.2	56.0	57.8	59.6	61.3	63.1	64.9	66.7	68.5	70.2
11	53.8	55.6	57.3	59.1	60.8	62.6	64.3	66.1	67.9	69.6
12	53.7	55.5	57.3	59.0	60.8	62.5	64.3	66.0	67.8	69.5
13	53.7	55.4	57.2	58.9	60.7	62.4	64.2	65.9	67.7	69.4

Depth : Metres. }	4000	4100	4200	4300	4400	4500	4600	4700	4800	4900
Pressure : Decibars. }	4071.0	4173.8	4276.6	4379.4	4482.3	4585.2	4688.2	4791.2	4894.3	4997.4
t °C.										
-1	72.4	74.2	76.0	77.8	79.6	81.4	83.1	84.9	86.7	88.5
0	72.4	74.2	76.0	77.7	79.5	81.3	83.1	84.9	86.6	88.4
1	72.3	74.1	75.9	77.7	79.4	81.2	83.0	84.8	86.6	88.3
2	72.3	74.0	75.8	77.6	79.4	81.1	82.9	84.7	86.5	88.2
3	72.2	74.0	75.7	77.5	79.3	81.1	82.8	84.6	86.4	88.1
4	72.1	73.9	75.6	77.4	79.2	80.9	82.7	84.4	86.2	88.0
5	72.0	73.8	75.5	77.3	79.0	80.8	82.6	84.3	86.1	87.8

TABLE 4—continued.

Depth : Metres. }	5000	5100	5200	5300	5400	5500	5600	5700	5800	5900
Pressure : Decibars. }	5100.6	5203.8	5307.1	5410.4	5513.7	5617.1	5720.5	5824.0	5927.5	6031.1
t °C.										
-1	90.3	92.1	93.8	95.6	97.4	99.2	100.9	102.7	104.5	106.3
0	90.1	92.0	93.8	95.5	97.3	99.1	100.8	102.6	104.4	106.1
1	90.1	91.9	93.7	95.4	97.2	99.0	100.7	102.5	104.3	106.0
2	90.0	91.8	93.5	95.3	97.0	98.8	100.5	102.3	104.1	105.8
3	89.8	91.7	93.5	95.2	96.9	98.7	100.4	102.2	103.9	105.6
4	89.7	91.5	93.2	95.0	96.7	98.5	100.2	101.9	103.7	105.4

Depth : Metres. }	6000	6100	6200	6300	6400	6500	6600	6700	6800	6900
Pressure : Decibars. }	6134.7	6238.4	6342.1	6445.9	6549.7	6653.5	6757.4	6861.3	6965.3	7069.3
t °C.										
-1	108.0	109.8	111.6	113.3	115.1	116.8	118.6	120.4	122.1	123.9
0	107.9	109.6	111.4	113.1	114.9	116.7	118.4	120.2	121.9	123.7
1	107.8	109.5	111.3	113.0	114.7	116.5	118.2	120.0	121.7	123.4
2	107.6	109.3	111.0	112.8	114.5	116.2	118.0	119.7	121.4	123.2
3	107.4	109.1	110.8	112.6	114.3	116.0	117.7	119.5	121.2	122.9
4	107.2	108.9	110.6	112.3	114.0	115.8	117.5	119.2	120.9	122.6

Depth : Metres. }	7000	7100	7200	7300	7400	7500	7600	7700	7800	7900
Pressure : Decibars. }	7173.4	7277.5	7381.6	7485.8	7590.0	7694.3	7798.6	7903.0	8007.4	8111.8
t °C.										
-1	125.6	127.4	129.1	130.8	132.6	134.3	136.1	137.8	139.5	141.3
0	125.4	127.1	128.9	130.6	132.3	134.1	135.8	137.5	139.2	141.0
1	125.2	126.9	128.6	130.3	132.0	133.8	135.5	137.2	138.9	140.6
2	124.9	126.6	128.3	130.0	131.7	133.4	135.2	136.8	138.6	140.3
3	124.6	126.3	128.0	129.7	131.4	133.1	134.8	136.5	138.2	139.9
4	124.4	126.1	127.7	129.4	131.1	132.8	134.5	136.2	137.9	139.6

Depth : Metres. }	8000	8100	8200	8300	8400	8500	8600	8700	8800	8900
Pressure : Decibars. }	8216.3	8320.9	8425.4	8530.1	8634.8	8739.5	8844.2	8949.0	9053.9	9158.8
t °C.										
-1	143.0	144.7	146.4	148.1	149.9	151.6	153.3	155.0	156.7	158.4
0	142.7	144.7	146.1	147.8	149.5	151.2	152.9	154.6	156.3	158.0
1	142.3	144.0	145.7	147.4	149.1	150.8	152.5	154.2	155.9	157.6
2	142.0	143.7	145.4	147.0	148.7	150.4	152.1	153.8	155.5	157.2
3	141.6	143.3	145.0	146.7	148.4	150.0	151.7	153.4	155.1	156.8
4	141.3	143.0	144.6	146.3	148.0	149.6	151.3	153.0	154.7	156.3

TABLE 4—continued.

Depth : Metres. }	9000	9100	9200	9300	9400	9500	9600	9700	9800	9900
Pressure : Decibars. }	9263.7	9368.7	9473.7	9578.7	9683.8	9789.0	9894.2	9999.4	—	—
t °C.										
-1	160.1	161.8	163.5	165.1	166.8	168.5	170.2	171.9	—	—
0	159.7	161.4	163.1	164.7	166.4	168.1	169.7	171.4	—	—
1	159.3	160.9	162.6	164.3	165.9	167.6	169.3	171.0	—	—
2	158.8	160.5	162.2	163.8	165.5	167.2	168.9	170.5	—	—
3	158.4	160.1	161.7	163.4	165.1	166.7	168.4	170.0	—	—
4	158.0	159.7	161.3	163.0	164.6	166.3	167.9	169.6	—	—

TABLE 5.

Corrections to Table 4 on account of the variation of Gravity with Latitude.

The Corrections are subtractive between the Equator and 45° Lat. and additive between 45° Lat. and the corresponding Pole.

Latitude, N. or S. }	0°	10°	20°	30°	40°	50°	60°	70°	80°	90°
Depth, Metres.										
0	0	0	0	0	0	0	0	0	0	0
1000	-0.1	-0.1	0.0	0.0	0	0	0.0	0.0	0.1	0.1
2000	-0.1	-0.1	-0.1	-0.1	0	0	0.1	0.1	0.1	0.1
3000	-0.1	-0.1	-0.1	-0.1	0	0	0.1	0.1	0.1	0.1
4000	-0.2	-0.2	-0.1	-0.1	0	0	0.1	0.1	0.2	0.2
5000	-0.2	-0.2	-0.1	-0.1	0	0	0.1	0.1	0.2	0.2
6000	-0.3	-0.3	-0.2	-0.1	0	0	0.1	0.2	0.3	0.3
7000	-0.3	-0.3	-0.3	-0.2	-0.1	0.1	0.2	0.3	0.3	0.3
8000	-0.4	-0.4	-0.3	-0.2	-0.1	0.1	0.2	0.3	0.4	0.4
9000	-0.4	-0.4	-0.3	-0.2	-0.1	0.1	0.2	0.3	0.4	0.4
10000	-0.5	-0.4	-0.3	-0.2	-0.1	0.1	0.2	0.3	0.4	0.5

TABLE 6.
Velocity of Sound in Fresh and Sea Water under Atmospheric Pressure.
(Metres per Second.)

Sal. =	Pure water	6.28	12.47	17.43	18.68	19.92	21.17	22.41	23.66	24.90	26.14	27.38	28.63	29.87	31.11	32.36	33.60	34.85	36.08	37.32	38.57	39.79	41.04
$\sigma_0 =$	— .1324	5	10	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33
°C.																							
— 2°	1389.4	1398	1406	1412.9	1414.6	1416.2	1417.8	1419.5	1421.1	1422.8	1424.4	1426.1	1427.7	1429.4	1431.0	1432.6	1434.3	1435.9	1437.6	1439.2	1440.8	1442.5	1444.1
— 1°	1394.4	1403	1411	1417.8	1419.4	1421.0	1422.6	1424.2	1425.8	1427.4	1429.1	1430.7	1432.4	1434.1	1435.7	1437.3	1439.0	1440.6	1442.2	1443.9	1445.5	1447.2	1448.9
0°	1399.3	1408	1416	1422.5	1424.1	1425.7	1427.3	1429.0	1430.6	1432.3	1433.9	1435.5	1437.1	1438.7	1440.4	1442.0	1443.6	1445.2	1446.9	1448.5	1450.1	1451.7	1453.4
1°	1404.1	1413	1421	1427.1	1428.8	1430.4	1432.0	1433.6	1435.3	1436.9	1438.5	1440.1	1441.7	1443.3	1444.9	1446.6	1448.2	1449.8	1451.4	1453.0	1454.6	1456.2	1457.8
2°	1408.9	1417	1425	1431.7	1433.3	1435.0	1436.6	1438.2	1439.8	1441.4	1443.0	1444.6	1446.2	1447.8	1449.4	1451.0	1452.6	1454.2	1455.8	1457.4	1459.0	1460.6	1462.2
3°	1413.6	1422	1430	1436.2	1437.8	1439.5	1441.1	1442.7	1444.3	1445.9	1447.5	1449.0	1450.6	1452.2	1453.8	1455.4	1457.0	1458.6	1460.2	1461.8	1463.3	1464.9	1466.5
4°	1418.3	1426	1434	1440.7	1442.3	1443.9	1445.4	1447.0	1448.6	1450.2	1451.8	1453.4	1455.0	1456.5	1458.1	1459.7	1461.3	1462.9	1464.4	1466.0	1467.6	1469.2	1470.7
5°	1422.8	1431	1439	1445.0	1446.6	1448.2	1449.8	1451.3	1452.9	1454.5	1456.1	1457.6	1459.2	1460.8	1462.3	1463.9	1465.5	1467.0	1468.6	1470.2	1471.7	1473.3	1474.9
6°	1427.3	1435	1443	1449.3	1450.8	1452.4	1454.0	1455.5	1457.1	1458.7	1460.2	1461.8	1463.4	1464.9	1466.4	1468.0	1469.6	1471.1	1472.7	1474.2	1475.8	1477.3	1478.9
7°	1431.7	1439	1447	1453.4	1455.0	1456.6	1458.1	1459.7	1461.2	1462.8	1464.3	1465.8	1467.4	1468.9	1470.5	1472.0	1473.6	1475.1	1476.6	1478.2	1479.7	1481.3	1482.8
8°	1436.0	1444	1451	1457.5	1459.0	1460.6	1462.1	1463.7	1465.2	1466.8	1468.3	1469.8	1471.3	1472.9	1474.4	1475.9	1477.5	1479.0	1480.5	1482.1	1483.6	1485.1	1486.7
9°	1440.2	1448	1455	1461.5	1463.0	1464.6	1466.1	1467.6	1469.1	1470.7	1472.2	1473.7	1475.2	1476.7	1478.2	1479.8	1481.3	1482.8	1484.3	1485.8	1487.3	1488.9	1490.4
10°	1444.3	1452	1459	1465.4	1466.9	1468.4	1469.9	1471.4	1473.0	1474.5	1476.0	1477.5	1479.0	1480.5	1482.0	1483.5	1485.0	1486.5	1488.0	1489.5	1491.0	1492.5	1494.0
11°	1448.3	1455	1463	1469.2	1470.7	1472.2	1473.7	1475.2	1476.7	1478.2	1479.7	1481.2	1482.7	1484.2	1485.7	1487.1	1488.6	1490.1	1491.6	1493.1	1494.6	1496.0	1497.5
12°	1452.1	1459	1467	1472.9	1474.4	1475.6	1477.3	1478.8	1480.3	1481.8	1483.3	1484.8	1486.3	1487.7	1489.2	1490.7	1492.2	1493.6	1495.1	1496.6	1498.0	1499.5	1500.9
13°	1455.9	1463	1471	1476.5	1478.0	1479.4	1480.9	1482.4	1483.9	1485.3	1486.8	1488.3	1489.8	1491.2	1492.7	1494.1	1495.6	1497.1	1498.5	1500.0	1501.4	1502.8	1504.3
14°	1459.6	1467	1474	1480.0	1481.5	1482.9	1484.4	1485.8	1487.3	1488.8	1490.2	1491.7	1493.2	1494.6	1496.1	1497.5	1498.9	1500.4	1501.8	1503.3	1504.7	1506.1	1507.5
15°	1463.2	1470	1478	1483.4	1484.9	1486.3	1487.8	1489.2	1490.7	1492.1	1493.6	1495.0	1496.5	1497.9	1499.3	1500.8	1502.2	1503.6	1505.0	1506.5	1507.9	1509.3	1510.7
16°	1466.7	1474	1481	1486.8	1488.2	1489.6	1491.1	1492.1	1493.9	1495.4	1496.8	1498.2	1499.7	1501.1	1502.5	1503.9	1505.5	1506.8	1508.2	1509.6	1511.0	1512.4	1513.8
17°	1470.0	1477	1484	1490.0	1491.4	1492.9	1494.3	1495.7	1497.1	1498.5	1500.0	1501.4	1502.8	1504.2	1505.6	1507.0	1508.4	1509.8	1511.2	1512.6	1514.0	1515.4	1516.8
18°	1473.3	1480	1487	1493.2	1494.6	1496.0	1497.4	1498.8	1500.2	1501.6	1503.0	1504.4	1505.8	1507.2	1508.6	1510.0	1511.4	1512.8	1514.2	1515.6	1517.0	1518.3	1519.7
19°	1476.5	1484	1491	1496.2	1497.6	1499.0	1500.4	1501.8	1503.2	1504.6	1506.0	1507.4	1508.8	1510.2	1511.6	1512.9	1514.3	1515.7	1517.1	1518.5	1519.8	1521.2	1522.6
20°	1479.5	1487	1494	1499.2	1500.6	1502.0	1503.4	1504.8	1506.2	1507.5	1508.9	1510.3	1511.7	1513.0	1514.4	1515.8	1517.1	1518.5	1519.9	1521.2	1522.6	1524.0	1525.4
21°	1482.5	1490	1496	1502.1	1503.5	1504.9	1506.3	1507.6	1509.0	1510.4	1511.7	1513.1	1514.4	1515.8	1517.2	1518.5	1519.9	1521.2	1522.6	1524.0	1525.3	1526.7	1528.0
22°	1485.3	1493	1499	1504.9	1506.3	1507.7	1509.0	1510.4	1511.8	1513.1	1514.5	1515.8	1517.1	1518.5	1519.8	1521.1	1522.5	1523.8	1525.1	1526.5	1527.8	1529.2	1530.5
23°	1488.0	1495	1502	1507.7	1509.0	1510.4	1511.7	1513.1	1514.4	1515.8	1517.1	1518.4	1519.8	1521.1	1522.5	1523.8	1525.1	1526.5	1527.8	1529.2	1530.5	1531.9	1533.3
24°	1490.7	1498	1505	1510.3	1511.7	1513.0	1514.3	1515.7	1517.0	1518.4	1519.7	1521.0	1522.4	1523.7	1525.0	1526.3	1527.7	1529.0	1530.4	1531.7	1533.0	1534.4	1535.7
25°	1493.2	1501	1508	1512.9	1514.2	1515.6	1516.9	1518.2	1519.5	1520.9	1522.2	1523.5	1524.9	1526.2	1527.5	1528.8	1530.2	1531.5	1532.8	1534.1	1535.4	1536.8	1538.1
26°	1495.6	1503	1510	1515.4	1516.7	1518.0	1519.4	1520.7	1522.0	1523.3	1524.7	1526.0	1527.3	1528.6	1530.0	1531.3	1532.6	1533.9	1535.2	1536.5	1537.8	1539.1	1540.4
27°	1498.0	1506	1513	1517.8	1519.2	1520.5	1521.8	1523.1	1524.4	1525.7	1527.1	1528.4	1529.7	1531.0	1532.3	1533.6	1534.9	1536.2	1537.5	1538.8	1540.1	1541.4	1542.7
28°	1500.2	1508	1515	1520.2	1521.5	1522.9	1524.2	1525.5	1526.8	1528.1	1529.4	1530.7	1532.0	1533.3	1534.6	1535.9	1537.2	1538.5	1539.8	1541.1	1542.4	1543.7	1545.0
29°	1502.3	1511	1517	1522.5	1523.9	1525.2	1526.5	1527.8	1529.1	1530.4	1531.7	1533.0	1534.3	1535.6	1536.9	1538.2	1539.5	1540.8	1542.1	1543.3	1544.6	1545.9	1547.2
30°	1504.4	1513	1520	1524.8	1526.1	1527.4	1528.8	1530.1	1531.4	1532.7	1534.0	1535.3	1536.6	1537.8	1539.1	1540.4	1541.7	1543.0	1544.2	1545.5	1546.8	1548.1	1549.3

TABLE 7.

1.	2.	3.	4.	5.	6.	7.	8.	9.	10.
Depth, Metres.	t° C.	Salinity per thousand.	Horizontal Velocity, m./sec.	Mean horizontal velocity in each 200 m. layer.	Sounding velocity from 5.	Mean temperature in 200 m. and 1,000 m. layers.	Mean salinity in 200 m. and 1,000 m. layers.	Mean velocity in layers from 7 and 8.	Sounding velocity from 9, m./sec.
0	23.91	37.27	1531.4	1527.6	—	21.94	37.11	1527.1	
50	22.89	37.19	1529.6						
100	21.83	37.17	1527.7						
200	19.14	36.80	1521.9						
400	15.19	36.08	1512.8	1517.4	1527.6	17.16	36.44	1517.5	1527.1
600	11.74	35.61	1504.4	1508.6	1522.5	13.46	35.84	1508.8	1522.3
800	9.79	35.39	1500.8	1502.6	1517.9	10.76	35.50	1502.6	1517.8
1000	7.49	35.17	1495.6	1498.2	1514.1	8.64	35.28	1498.3	1514.0
1200	6.19	35.10	1494.0	1494.8	1510.9	6.84	35.13	1494.7	1510.9
1400	5.53	35.09	1494.9	1494.4	1508.2	5.86	35.09	1494.2	1508.2
1600	5.11	35.09	1496.9	1495.9	1506.2	5.32	35.09	1495.6	1506.2
1800	4.96	35.09	1499.9	1498.4	1505.0	5.03	35.09	1498.1	1504.9
2000	4.81	35.09	1502.8	1501.3	1504.2	4.89	35.09	1501.2	1504.1
2200	4.66	35.09	1505.8	1504.3	1503.9	4.43	35.09	1510.1	1503.8
2400	4.51	35.09	1508.8	1507.3	1504.0				1503.9
2600	4.35	35.09	1511.7	1510.2	1504.2				(1504.2)
2800	4.20	35.09	1514.7	1513.2	1504.7				(1504.6)
3000	4.05	35.09	1517.7	1516.2	1505.3	3.67	35.09	1524.9	(1505.2)
3200	3.90	35.09	1520.6	1519.1	1506.0				(1505.9)
3400	3.75	35.09	1523.5	1522.2	1506.8				(1506.6)
3600	3.60	35.09	1526.3	1524.9	1507.8				(1507.5)
3800	3.45	35.09	1529.3	1527.8	1508.7	2.92	35.09	1540.2	(1508.5)
4000	3.30	35.09	1532.3	1530.8	1509.7				(1509.5)
4200	3.15	35.09	1535.2	1533.8	1510.8				1510.7
4400	2.99	35.09	1538.2	1536.7	1511.9				(1511.8)
4600	2.84	35.09	1541.0	1539.6	1513.0	2.92	35.09	1540.2	(1512.9)
4800	2.69	35.09	1543.9	1542.4	1514.2				(1514.1)
5000	2.54	35.09	1546.8	1545.4	1515.3				(1515.3)
5108	2.46	35.09	1548.2	1547.5	1516.5				1516.6

PART III.

METHODS USED IN PREPARING THE TABLES OF VELOCITY.

The velocity of sound-waves in water can be determined either by direct measurement over long distances or indirectly by calculation from other properties which can be investigated under laboratory conditions.

One of the indirect methods, and probably the best, is that used for the present tables and is by calculation from the formula

$$V = \sqrt{\frac{dp}{d\rho}}$$

where V = velocity,

p = pressure

and ρ = density.

For convenience in the arithmetical work this may be written in the form

$$V = \sqrt{\frac{\gamma}{\rho\kappa}}$$

where κ = the true compressibility, $-\frac{1}{v} \frac{dv}{dp}$, v being volume, and

γ is the ratio of the specific heats of the water at constant pressure and constant volume. This last factor is introduced to allow for the fact that a sound-wave is a wave of compression and that it heats the water as it passes through it. Since water is a very bad conductor of heat the wave has passed away from each heated portion before it has had time to cool down to the general temperature of the surroundings and therefore travels in a medium which is warmer than that for which the velocity is calculated. The only exception to this is at the point where the water is at its maximum density; here γ is unity.

Before the numerical value of these quantities can be found it is necessary to know with regard to the water

(1) The temperature.

(2) The salinity, or alternatively the density at $0^{\circ}/4^{\circ}$ C., since water can be described equally well in either way. In practical oceanography it is usual to determine the chlorine by chemical analysis and then to take out the corresponding salinity and density from *Knudsen's Hydrographical Tables*, 1901. The density at $0^{\circ}/4^{\circ}$ C. is commonly contracted by subtracting unity and then multiplying by 1,000; this form is called σ_0 . For example, $\sigma_0 = 28$ means that density $0^{\circ}/4^{\circ}$ C. = 1.028.

(3) The pressure, which depends upon the depth, the density of the overlying water, and the acceleration of gravity, and therefore upon the latitude.

Then

$$\rho = \frac{\rho_0}{1 - p\mu}$$

where p is pressure in bars and μ is the mean compressibility per bar between pressure = 0 and pressure = p . One bar is one million dynes per square centimetre and is nearly one atmosphere.

ρ_0 = density at atmospheric pressure and is calculated from the chlorine or salinity by *Knudsen's Tables*, in which the corresponding contracted form is σ_t .

μ , the mean compressibility, has been determined by V. W. Ekman.* He used Amagat's data on the compressibility of pure water as the foundation of his measurements and put his results in the form—

$$\begin{aligned}\mu \times 10^8 = & \frac{4886}{1 + .000183p} - \left[227 + 28.33t - 0.551t^2 + 0.004t^3 \right] \\ & + \frac{p}{1000} \left[105.5 + 9.50t - 0.158t^2 \right] - \frac{1.5p^2t}{1000000} \\ & - \frac{\sigma_0 - 28}{10} \left[147.3 - 2.72t + 0.04t^2 - \frac{p}{1000} \left(32.4 - 0.87t + 0.02t^2 \right) \right] \\ & + \frac{(\sigma_0 - 28)^2}{100} \left[4.5 - 0.1t - \frac{p}{1000} (1.8 - 0.06t) \right].\end{aligned}$$

κ , the true compressibility,

$$= -\frac{1}{v} \frac{dv}{dp} = \frac{\mu + p \frac{d\mu}{dp}}{1 - p\mu} \times 10^{-6}.$$

γ has not yet been determined directly, but it can be shown on thermodynamical grounds that

$$\gamma = \frac{C_p}{C_v} = \frac{C_p}{C_p - \frac{T e^2}{\rho \kappa J}}$$

where C_p = specific heat at constant pressure in calories per gram per degree C.

C_v = specific heat at constant volume,

J = the mechanical equivalent of heat,

T = the absolute temperature,

e = the coefficient of thermal expansion, determined by Ekman and Knudsen,

and the other symbols have the same meaning as before.

C_p , the specific heat of sea water at constant pressure, has been determined by Thoulet and Chevallier.† It has here been assumed to vary with temperature in the same way as the specific heat of pure water. It decreases considerably with increase of pressure, as is shown by the following thermodynamical relation :—

$$\frac{\delta C_p}{\delta p} = -10^5 \frac{T}{J} \frac{\delta^2 v}{\delta t^2} = -10^5 \frac{T}{\rho J} \left(\frac{\delta e}{\delta t} + e^2 \right)$$

where p is in decibars and v is specific volume. The necessary correction has been applied throughout; in extreme cases it may affect the velocity to the extent of half a metre per second.

The constants and the velocities have been calculated directly for each five degrees of temperature and each five units in the

* *Pub. de Circ. of the Cons. Perm. Internat. pour l'Explor. de la Mer*, No. 43.

† Krummel's "*Oceanographie*," 1907 edition, i., 279.

value of σ_0 from $\sigma_0 = 5$ upwards and then interpolated for units by means of an interpolation formula, using third and sometimes fourth differences. Independent calculations have been made at -2°C. and -1°C. ; this has been made necessary by the temperature of maximum density which falls in this region in the case of sea water. The calculations for pure water have been made at each degree up to 6°C. by means of an independent formula for its compressibility given by Ekman.

These calculations give the velocity referred to pressure, but for practical purposes it is necessary that they should be referred to depth. Ekman has published* tables showing the depth at which a given pressure occurs in any latitude, and these have been used to transform the corrections on account of pressure to the corrections for depth and latitude given in Tables 4 and 5. The pressures corresponding to the depths are given at the head of each column so that the correction can be taken out with regard to pressure if desired. It has been assumed in making the change that the density of the ocean is everywhere that corresponding to 0°C. and a salinity of 34.85 per thousand as has been pointed out on p. 11; a correction for departure from this ideal condition should, strictly, be applied, which might amount in an extreme case to 10 decibars, corresponding to 0.2 m./sec., but since high temperatures in the depths are generally associated with high salinities, and these oppose one another in their effects, it has been neglected.

Sounding tables and tables of the velocity of sound have been published by various investigators; they differ in methods and results from the present ones, and in some cases the difference is important.

Dr. Maurer† has obtained the necessary values of the compressibility by taking the differences in specific volume of sea-water from Bjerknes' tables; he has not made any use of Ekman's formula, which is the basis of Bjerknes' work. The resulting velocities would not, therefore, attain a high degree of accuracy. He has also failed to take account of the fact that the ratio of the specific heats is not generally unity. The calculations are made for water of salinity 32.3 per thousand, in which M. Marti made his experimental determinations at Cherbourg; they show large differences from the results of observation.

Dr. Schumacher‡ has used Bjerknes' tables for the density and Ekman's formula for the compressibility. Since the tables depend on the formula, it might have been simpler to have combined the two quantities in one expression, especially as they occur again in the same form in the expression for the ratio of the specific heats. Dr. Schumacher has, however, neglected this ratio on the ground of its unimportance and quotes the value for distilled water at 13°C. as 1.001 under atmospheric pressure.

* *Pub. de Circ.*, No. 49.

† "*Ann. d. Hydr.*," 1924, p. 86.

‡ "*Ann. d. Hydr.*," 1924, p. 93.

The true value is more nearly 1.002, and in any case he has overlooked the fact that its value depends upon the square of the coefficient of expansion, which increases rapidly at high temperatures and pressures. The most important difference, however, between his methods and those used for the preparation of these tables is that he has used the mean compressibility, μ instead of the true compressibility

$$\frac{\mu + p \frac{d\mu}{dp}}{1 - p\mu}.$$

The difference is zero at the surface, but becomes large at great pressures. Ekman's μ is the total compression which unit volume would undergo if the pressure were raised from that of the atmosphere to p bars, divided by p . The true compressibility κ at any pressure is the compression per unit volume per bar and the mean of the true compressibilities over any range is not the same as the corresponding value of μ .

The most complete set of tables hitherto published is that by Commander N. H. Heck and Ensign Jerry H. Service of the U.S. Coast and Geodetic Survey.* They, too, have used Bjerknes' specific volume tables for the compressibility, and the consequent rounding off of the last figures has been the cause of some inaccuracies. At 0° C. at the surface, for instance, they find that $\frac{dv}{dp}$ for water of 35 salinity per thousand is 0.000045, while Ekman's formula shows that 0.00004658 is the true value. The result is that they get for the corresponding velocity 1450 m./sec., compared with 1445.5 in the present tables. They have calculated the adiabatic correction for the ratio of the specific heats, but express some doubt as to the advisability of using it and have therefore printed it in a separate table.

COMPARISON OF OBSERVED AND CALCULATED VELOCITIES.

Colladon and Sturm found that the velocity of sound in the Lake of Geneva at a temperature of 8.1° C. was 1435 m./sec.† The calculated velocity is 1436.4, in fairly good agreement. Martini, on the other hand, in 1888 found for fresh water 1399 m./sec. at 4° C. and 1457 m./sec. at 25° C. against the calculated velocities 1418 m./sec. and 1493 m./sec.

The result of a careful series of measurements made by M. Marti in 1919 at Cherbourg for the French Admiralty was that the velocity at 15° C. in water of density 1.026 at a pressure of one atmosphere is 1504.15 m./sec.; the agreement with the calculated value, 1500.9, is not so good as could be wished.

* *Special Publication, No. 108, Washington, 1924.*

† Kaye and Laby's *Tables*.

Dr. A. B. Wood, Commander H. E. Browne, R.N., and Mr. C. Cochrane* made a very careful series of determinations under summer and winter conditions off the Goodwin Sands at a mean depth of 16 fathoms. Their results are given below, together with the theoretical velocities; the latter are corrected for the depth.

Temperature =	6° C.	7° C.	16·95° C.
σ_0	= 28·21	28·29	28·14
V observed	= 1474·0 m./sec.	1477·3 m./sec.	1510·4 m./sec.
V theory	= 1472·0 m./sec.	1476·1 m./sec.	1510·4 m./sec.
Diff.	= - 2·0	- 1·2	0·0

Owing to difficulties arising from stormy weather in the winter Dr. Wood does not consider the two sets at low temperatures so reliable as those at 16·95° C., when the agreement was perfect, but he does not estimate that the uncertainty was greater than 0·6 m./sec. and 0·5 m./sec.

Heck and Service quote the following experimental determinations :—

E. B. Stephenson found at $-0\cdot3^\circ$ C. and in salinity 33·5 per thousand a velocity of $1453 \pm 1\cdot5$ m./sec. at the surface; these tables give 1442·2 m./sec.

Dr. E. A. Eckhardt found at the surface, at 13° C. and salinity 33·5, 1492 m./sec., compared with 1495·5 m./sec. according to these tables.

The *U.S.S. Guide* as the mean of several observations obtained 1495 m./sec. at 14° C. and salinity 33·5 near the surface. The theoretical velocity is 1498·6 m./sec.

The accuracy of these tables depends upon the following constants :—

(1) *The density and coefficient of thermal expansion of sea-water of varying salinity and temperature under atmospheric pressure.* The authority for the values of these quantities is *Knudsen's Hydrographical Tables*, 1901, and they are probably known more accurately than any other quantity used in the calculations.

(2) *The compressibility of water.* On account of the difficulty of measuring high pressures the accuracy probably falls off as the depth increases, but there are no independent determinations which can be satisfactorily used as a check. Ekman's formula is founded on observations made upon pure water and upon sea-water of 31·13 and 38·53 salinity, and consequently the accuracy of the tables is less for water of salinity less than 30 per thousand than for saltier water; the error due to this uncertainty probably varies between 0·5 m./sec. low at 10° C. and 1·6 m./sec. high at 30° C. Low salinities, however, never extend to great depths and the resulting errors in soundings will be small. The table for pure water is probably of a high order of accuracy. Similar errors will occur in the case of water of over 38·5

* *Proc. Roy. Soc.*, 103A, pp. 284–303, May 3, 1923.

salinity, but they will be less important. No measurements of compressibility were made at temperatures over 20° C., and accordingly doubt will attach to the velocities in warmer waters. This will be of no importance in vertical sounding, since higher temperatures do not occur in the depths, but it makes it impossible to use the tables at present for accurate measurement of horizontal ranges in hot climates.

(3) *The specific heat at constant pressure.* The values used are probably accurate enough for the calculation of surface velocities. The calculation of the decrease with increasing pressure depends upon the square of the coefficient of thermal expansion under pressure and therefore upon the formula for compressibility. This is probably the least accurate part of the calculations.

The summer experiments off the Goodwin Sands appear to be the most satisfactory from the point of view of the general accuracy with which the range, temperature and salinity were known, and it is remarkable that here there was no difference between theory and observation as great as one-tenth of a metre per second—roughly one part in fifteen thousand. It follows, therefore, that the tables may be considered accurate for water of about 17° C. and 35 salinity per thousand—conditions which, unfortunately, do not extend to any considerable depth. There is, however, no reason to consider that Ekman's formula falls off in accuracy at lower temperatures, especially since he had in view particularly the conditions which generally prevail in the depths—that is, temperatures from, say, 8° C. downwards. The results of experiments at 6° C. under winter conditions are not in such good accord with theory, the differences reaching 2 m./sec.

The American determinations of the velocity already referred to show wider deviations at -0.3° C., but at 13° C. and 14° C. they are only 3.5 m./sec. less than theory demands.

In the absence of a larger number of direct control measurements it is difficult to come to any very definite conclusion as to the accuracy of these tables, but it seems probable that they are reliable to two metres per second at temperatures up to about 22° C., in water such as normally occurs in the open sea, and at depths not greater than 2,000 fathoms or 4,000 metres, and that the accuracy is much greater in shallow water.